

Restrained Dark $U(1)_d$ at Low Energies

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We investigate a spontaneously broken $U(1)_d$ gauge symmetry with a muon-specific dark Higgs. Our first goal is to verify how the presence of a new dark Higgs, ϕ , and a dark gauge boson, V , can simultaneously face the anomalies from the muon magnetic moment and the proton charge radius. Secondly, by assuming that V must decay to an electron-positron pair, we explore the corresponding parameter space determined with the low energy constraints coming from $K \rightarrow \mu X$, electron $(g-2)_e$, $K \rightarrow \mu\nu_\mu e^+e^-$, $K \rightarrow \mu\nu_\mu\mu^+\mu^-$, $\tau \rightarrow \nu_\tau\mu\nu_\mu e^+e^-$. We focus in the scenario where the V mass is below $\sim 2m_\mu$ and the ϕ mass runs from few MeV till 250 MeV, with V-photon mixing of the order $\sim \mathcal{O}(10^{-3})$. Among weak process at low energies, we check the influence of the new light vector on kaon decays as well as on the scattering $e^+e^- \rightarrow \mu^+\mu^-e^+e^-$ and discuss the impact of the dark Higgs on $e^+e^- \rightarrow \mu^+\mu^-\mu^+\mu^-$. Finally, we consider contributions of the V-photon mixing in the decays $\pi^0 \rightarrow \gamma e^+e^-$, $\eta \rightarrow \gamma e^+e^-$, $\rho \rightarrow \pi e^+e^-$, $K^* \rightarrow K e^+e^-$ and $\phi(1020) \rightarrow \eta e^+e^-$.

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The dark matter abundance in the universe has stimulated numerous searches for the Standard Model (SM) extensions. Nevertheless, the true nature of the supposed new interaction is not understood yet and many models were suggested to explain it by assuming the existence of cosmologically stable particles, ranging in mass from below 1 GeV to above 1 TeV [1].

Moreover, there is a number of discrepancies between SM theoretical predictions and experimental results at energies below the kaon mass which might be signatures of new physics. The long lasting muon anomalous magnetic moment $(g-2)_\mu$ puzzle, for instance, is still present at 3.6σ level and as claimed by many authors [1–9], it can be explained by new dark bosons with the masses below 200 MeV. One more recent example is the discrepancy between the proton charge radius measured via the Lamb shift in atomic and muonic hydrogen (see e.g. [10–12]).

One of the proposals to explain the origin of such low energy puzzles is based on the spontaneously broken $U(1)_d$ gauge symmetry [1–9], introduced in the context of astrophysical anomalies (see e.g. [13, 14]). Its basic mechanism allows the gauge coupling to be $\sim O(10^{-3})$ and implies a kinetic mixing amplitude between the new gauge boson V and the photon field.

In decays with particles identified through the missing energy one might expect that some set of the invisible states is due to the existence of the coupled dark sector. Many experiments are devoted to search for weakly interacting particles [15–17] and one pioneering work in this direction was done by the authors of [18], who succeeded to put limits on the decay $BR(K \rightarrow \mu \text{ missing energy})$. By applying these bounds, the authors in [19] have found, for example, that the leptonic decay $K \rightarrow \mu\nu V$ is already very constraining on the V parameters.

In this paper we re-investigate a spontaneously broken $U(1)_d$ gauge model following the ideas presented in [3] and implementing additional constraints. First we assume that both the dark gauge boson, V , and the dark Higgs, ϕ , cannot be directly detected and assume that they both are present in the explanation of the proton size anomaly and kaon leptonic decays. We find a tension between the upper bounds on the decay width of the kaon leptonic decay and the proton size band for a specific range of relevant parameters. Secondly, we loose this prior restriction and since we are mainly interested in the low mass region, we continue to treat the dark Higgs as the muon-specific scalar, contributing to the missing mass and work in the scenario where V must decay to e^+e^- . The model will be further constrained by the BaBar additional observables: the uncertainty in $\Gamma(K \rightarrow \mu\nu_\mu e^+e^-)$, for $m_{ee} > 145$ MeV, the upper bound for $\tau \rightarrow \nu_\tau \mu\nu_\mu e^+e^-$ and by the electron anomalous magnetic moment $(g-2)_e$. When the dark Higgs mass is in the range $2m_\mu \leq m_\phi \leq (m_K - m_\mu)$, we derive bounds from the experimental upper bound on $\Gamma(K \rightarrow \mu\nu_\mu \mu^+\mu^-)$.

The analysis described above can be considered complementary to the recent BaBar result [16] on the search for a new neutral vector boson in the process $e^+e^- \rightarrow \mu^+\mu^-V$. Their result has placed very strong limits on the coupling constant of V , indicating that the presence of a massive vector state can be excluded in the range $0.212 - 10$ GeV.

Section I contains the description of the model we explore in our study. Sec. II is devoted to the derivation of bounds from low energy phenomenology. In Sec. III we discuss implications of this proposal on the low energy processes and Sec. IV contains the short summary of our results.

I. FRAMEWORK: DARK $U(1)_d$

The $U(1)_d$ gauge invariant Lagrangian under consideration is written by [3]:

$$\mathcal{L} = -\frac{1}{4}V_{\alpha\beta}V^{\alpha\beta} + |D_\mu\phi|^2 + \bar{\mu}_R i \not{D} \mu_R - \frac{\kappa}{2}V_{\alpha\beta}F^{\alpha\beta} - \bar{L}\mu_R H_{SM} \frac{\phi}{\Lambda} + h.c. \quad (1)$$

Here V is the gauge boson, neutral under the SM gauge group and charged under $U(1)_d$. The field ϕ is the dark Higgs with a condensate $\langle\phi\rangle = \frac{v_R}{\sqrt{2}}$. The covariant derivative $D_\alpha = \partial_\alpha + ig_R V_\alpha + ieQ_{EM}A_\alpha$ and κ is the mixing angle.

The muon mass is then introduced as $vv_R/(2\Lambda)$, while the SM-like Yukawa coupling is given by $v_R/(\sqrt{2}\Lambda)$. As asserted by the authors of [3], the proton charge radius phenomenology will favor the range of the new parameters such that the scale Λ can be at the weak scale. Moreover, the model given in (1) leads to gauge anomalies involving the photon and the vector V and in order to restore gauge invariance, it is mandatory to introduce new dynamical scalar degrees of freedom.

There are different ways to make this theory UV complete. For instance, a number of SM extensions with new vector-like fermions were constructed for this purpose [20–25]. In [26] it was suggested to extend the $SM \otimes U(1)_d$ by three right-handed neutrinos in order to generate neutrino masses. One last example was recently offered in [9] by the "lepton-specific" representation of a generic two Higgs doublet model in which the scalar sector contains the SM Higgs, an additional doublet and the dark ϕ .

The Lagrangian given in (1) leads to the following couplings of the new vector and scalar to fermions:

$$V \rightarrow -i\gamma^\mu(\bar{g}_V^\mu + \bar{g}_A^\mu\gamma_5), \quad \phi \rightarrow -ig_\phi \quad (2)$$

with the definitions

$$\bar{g}_V^\mu = e\kappa + \frac{g_R}{2}, \quad \bar{g}_A^\mu = \frac{g_R}{2}, \quad g_\phi = g_R \frac{m_\mu}{M_V} \quad (3)$$

In our analysis we choose to work with the set of parameters $(g_\phi, m_\phi, \kappa, M_V)$, by assuming $g_R = 2\lambda\kappa$ which, from the relation (3), leave us with:

$$\lambda = \frac{M_V}{m_\mu} \frac{g_\phi}{2\kappa}. \quad (4)$$

In the next section we will preferably consider specific choice of (λ, m_ϕ) since in the literature bounds and predictions are often presented for the space (M_V, κ) . We will also mention this combination in Sec. III.

II. LOW ENERGY PHENOMENOLOGY BOUNDS

One of main goals of the model presented in eq. (1) was to explain the proton size discrepancy [3]. It was first noticed by the authors of [19] that the $K \rightarrow \mu X$ decay, with X being a set of states seen only as missing energy, can give very strong constraints on the parameters of V [18]. Nevertheless, it was assumed that only an invisible vector state gives new contribution to this process. We first try to establish parameter space of (M_V, κ) which is allowed by the proton charge radius and the leptonic kaon decay, including the contributions of both vector and scalar dark bosons as missing mass.

The procedure described above can be summarised as:

- *Proton Charge Radius*

The measurement of the Lamb shift in muonic and atomic hydrogen ([27, 28]) has indicated a difference for the proton radius square, r_p^2 , which can be abbreviated to (for details see [3]):

$$\Delta r_p^2 = (r_p)_{e-p}^2 - (r_p)_{\mu-p}^2 = 0.060(12) \text{ fm}^2. \quad (5)$$

As discussed in [3], this discrepancy can be properly approached by the model of eq. (1) due to the mixing with the photon. Moreover, since the dark Higgs couples to muons only, the mass m_ϕ will remain free to adjust additional limits. Here we rewrite the theoretical corrections to the difference in eq.(5), following the notation of eq. (3):

$$\Delta r^2|_{e-H} = -\frac{6\kappa^2}{M_V^2}, \quad \Delta r^2|_{\mu-H} = -\frac{6\kappa^2(1 + \frac{\lambda}{e})}{M_V^2} f(aM_V), \quad (6)$$

where $a = (\alpha m_\mu m_p)^{-1}(m_p + m_\mu)$ is the $\mu - H$ Bohr radius, α is the fine-structure constant, and $f(z) = (z/(1+z))^4$. Therefore, for $aM_V \gg 1$ one can obtain the 2σ favourable region for the parameter κ , using the proton radius discrepancy given in eq. (6):

$$\kappa^2 = \frac{eM_V^2}{6\lambda} (\Delta r_p^2 \pm 2\sigma). \quad (7)$$

- *Muonic Kaon Decay $K \rightarrow \mu X$*

In the context of V and ϕ bremsstrahlung from μ , the result of analysis in ref. [18] can be converted to the upper bound:

$$\frac{\Gamma_{K \rightarrow \mu X}}{\Gamma_{K \rightarrow \mu \nu}} < 3.5 \times 10^{-6}, \quad 227.6 < m_X (\text{MeV}) < 302.2 \quad 90\% \text{ C.L.} \quad (8)$$

with

$$\Gamma_{K \rightarrow \mu X} = \Gamma_{K \rightarrow \mu \nu V} + \Gamma_{K \rightarrow \mu \nu \phi} \quad (9)$$

Note that in eq. (8) there is an experimental acceptance on the missing mass, m_X . The eq. (9) can be written as function of $(\kappa, M_V, \lambda, m_\phi)$.

In Fig. (1) we present the allowed parameter space (M_V, κ^2) obtained when the constraints from eqs. (7) and (8) are applied, for fixed values of (λ, m_ϕ) . On these particular examples the grey colour denotes the region excluded at the 90% C.L. by the bound on $BR(\Gamma(K \rightarrow \mu X))$, while the pink one denotes the region allowed by the proton size anomaly up to 2σ . We have checked if there is any region which can satisfy both conditions. For a large set of the (λ, m_ϕ) points we could not find any positive solution.

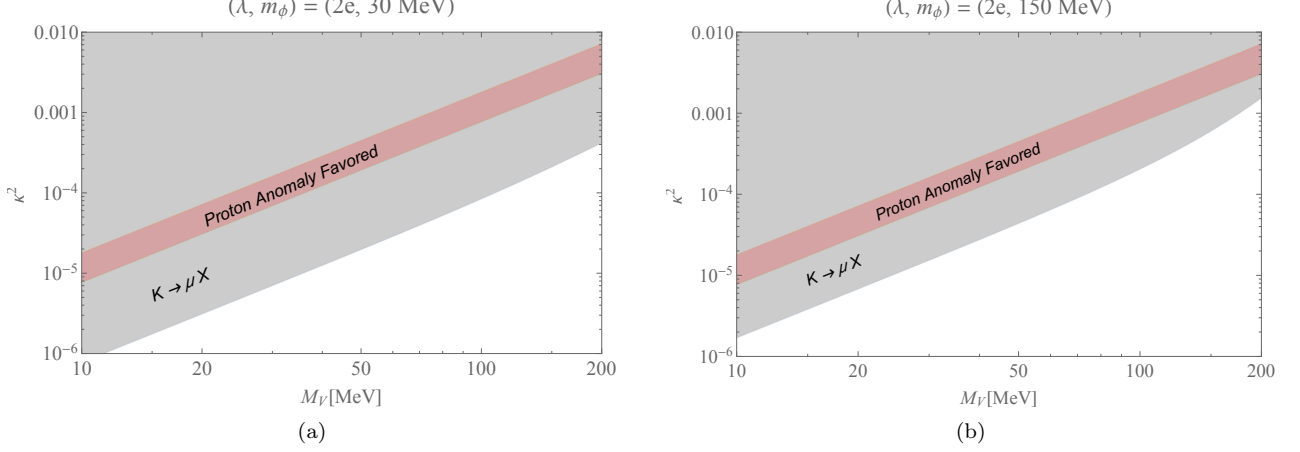


Figure 1: The (M_V, κ^2) parameter space fixed by the bounds from $K \rightarrow \mu X$ with the muonic V and ϕ bremsstrahlung (grey) along with the allowed area of the proton size anomaly (pink). The grey color marks the excluded region at the 90% C.L.

In this section we would like to illustrate how the muonic kaon decay itself is very restrictive. Once the dark Higgs is muon-specific, and we are mainly interested in the regime of small masses (bellow $2m_\mu$), we have to relax our first assumption and assume that the gauge boson V decays to electron-positron pair $V \rightarrow e^+e^-$. However, such Ve^+e^- interaction creates additional effects in a number of processes.

A. Muon Anomalous Magnetic Moment

The discrepancy between experimental results and the SM prediction for $(g-2)_\mu$ persists as an intriguing low energy puzzle in particle physics, currently being $\delta a_\mu^{\text{exp-SM}} = \Delta(g-2)_\mu/2 = 288(80) \times 10^{-11}$ [29].

The model of eq. (1) contributes at one-loop level with the three different contributions to $(g-2)_\mu$ - vector, axial-vector and the scalar one. The authors of [3] have noticed that within this framework there will be an enhancement of opposite sign to δa_μ , if compared to the pure vector case. Such feature might allow an overlap with the proton anomaly allowed region, since in the eq. (7) there is no dependence on m_ϕ . We can clearly see this feature through writing the complete expression below (see [30] and [31]):

$$\begin{aligned} \delta a_\mu &= (g_V)^2 I_V \left(\frac{M_V^2}{m_\mu^2} \right) + (g_A)^2 I_A \left(\frac{M_V^2}{m_\mu^2} \right) + (g_\phi)^2 I_\phi \left(\frac{m_\phi^2}{m_\mu^2} \right), \\ &= \kappa^2 \left[(e + \lambda)^2 I_V \left(\frac{M_V^2}{m_\mu^2} \right) + (\lambda)^2 I_A \left(\frac{M_V^2}{m_\mu^2} \right) + \left(\frac{2m_\mu \lambda}{M_V} \right)^2 I_\phi \left(\frac{m_\phi^2}{m_\mu^2} \right) \right] \\ &\equiv \kappa^2 F(\lambda, M_V, m_\phi). \end{aligned} \tag{10}$$

The full expression for the one-loop integrals $I_{V,A,\phi}$ can be found in [30]. We note that in the regime where the function $F(\lambda, M_V, m_\phi)$ is small the coupling κ can reach arbitrarily large values. This behaviour will be tested in the subsection II F.

B. Leptonic Kaon Decays

The first conclusion of our analysis is that the dark photon must decay to an electron-positron pair. This, however, does not mean that $K \rightarrow \mu X$ bound is not important anymore, since the scalar ϕ still takes a role as a missing mass.

The relation (3) accompanied by $g_R = 2\lambda\kappa$, will again produce the excluded region for (M_V, κ^2) , even being dependent on m_ϕ . Moreover, given the richness of kaon phenomenology, the new requirement applied on the additional channels involving V can produce even stronger bounds on the parameter space:

- $K^+ \rightarrow \mu^+ \nu_\mu e^+ e^-$

The branching ratio for this process is given in [29]:

$$\frac{\Gamma(K^+ \rightarrow \mu^+ \nu_\mu e^+ e^-)}{\Gamma_K} = 7.06(31) \times 10^{-8} \quad (m_{ee} > 145 \text{ MeV}). \quad (11)$$

The authors of [5] considered contributions of V via kinetic mixing with a radiated SM photon, as in refs. [12, 32, 33]. They also made a comparison between $K^+ \rightarrow \mu^+ \nu_\mu V \rightarrow \mu^+ \nu_\mu e^+ e^-$ and the QED background [34], having found that a new light vector boson, if it decays before leaving the detector, might produce bumps in the electron-positron invariant mass spectrum. The model presented in eq. (1), which we use, leads to a signal few orders of magnitude larger than the proposal of [5].

We assume that these NP corrections by itself should not be larger than 1σ of the result given in eq. (11). Using the narrow-width approximation and anticipating that the range for M_V is $145 \text{ MeV} < M_V < 2m_\mu$, where V can decay only to $e^+ e^-$, we can impose the following upper bound:

$$\Gamma(K^+ \rightarrow \mu^+ \nu_\mu V, V \rightarrow e^+ e^-) = \Gamma(K^+ \rightarrow \mu^+ \nu_\mu V) \times \text{Br}(V \rightarrow e^+ e^-). \quad (12)$$

Finally, since $\text{Br}(V \rightarrow e^+ e^-) = 1$ it becomes:

$$\frac{\Gamma(K^+ \rightarrow \mu^+ \nu_\mu V)}{\Gamma_K} < 3.1 \times 10^{-9}. \quad (13)$$

- $K^+ \rightarrow \mu^+ \nu_\mu \mu^+ \mu^-$

If the muon-specific dark Higgs has a mass larger than $2m_\mu$, the ϕ bremsstrahlung will be then followed by the decay $\phi \rightarrow \mu^+ \mu^-$. In this case the bound from the $K \rightarrow \mu X$ cannot be applied. The $K \rightarrow \mu X$ constraint should be replaced by the existing upper bound [29]:

$$\frac{\Gamma(K^+ \rightarrow \mu^+ \nu_\mu \mu^+ \mu^-)}{\Gamma_K} < 4.1 \times 10^{-7}, \quad 90\% \text{ C.L.} \quad (14)$$

The above expression can be useful in the region $2m_\mu < m_\phi < (m_K - m_\mu)$ and as we will find out in the section II F, if m_ϕ is close to $2m_\mu$ this constraint is equally powerful as one coming from $K \rightarrow \mu X$ (8). Since the dark Higgs interacts with muons only, its decay to $\mu^+ \mu^-$ is allowed now and $\text{Br}(\phi \rightarrow \mu^+ \mu^-) = 1$. Using the narrow-width approximation, we obtain:

$$\frac{\Gamma(K^+ \rightarrow \mu^+ \nu_\mu \phi)}{\Gamma_K} \times \text{Br}(\phi \rightarrow \mu^+ \mu^-) = \frac{\Gamma(K^+ \rightarrow \mu^+ \nu_\mu \phi)}{\Gamma_K} < 4.1 \times 10^{-7}, \quad 90\% \text{ C.L.} \quad (15)$$

C. Constraints from $\tau^+ \rightarrow \nu_\tau \mu^+ \nu_\mu e^+ e^-$

Within the SM, the $e^+ e^-$ pair in the process $\tau^+ \rightarrow \nu_\tau \mu^+ \nu_\mu e^+ e^-$ originates from the virtual photon or Z emission in the decay $\tau^+ \rightarrow \nu_\tau \mu^+ \nu_\mu$. Analogously to the previous case with $V \rightarrow e^+ e^-$, we can consider the upper bound to the ratio of this process by assuming that one can safely use the narrow-width approximation:

$$\frac{\Gamma(\tau \rightarrow \nu_\tau \mu \bar{\nu}_\mu V)}{\Gamma_\tau} \text{Br}(V \rightarrow e^+ e^-) = \frac{\Gamma(\tau \rightarrow \nu_\tau \mu \bar{\nu}_\mu V)}{\Gamma_\tau} < 3.6 \times 10^{-5}, \quad 90\% \text{ C.L.} \quad (16)$$

The differential decay rate for $\tau^+ \rightarrow \nu_\tau \mu^+ \nu_\mu e^+ e^-$ is given by:

$$\begin{aligned} d\Gamma_{\tau \rightarrow \nu_\tau \bar{\nu}_\mu V \mu} &= \frac{m_\tau^3}{256(2\pi)^6} |\mathcal{M}_{\tau \rightarrow \nu_\tau \bar{\nu}_\mu V \mu}|^2 \sqrt{\lambda(1, \delta_3, 0)} \sqrt{\lambda(\delta_2, \delta_\mu, \delta_V)} \\ &\times \frac{\sqrt{\lambda(\delta_3, \delta_2, 0)}}{\delta_2 \delta_3} d\delta_2 d\delta_3 dC_{\theta_2} dC_{\theta_3} d\phi, \end{aligned} \quad (17)$$

where we have assigned the momenta $\tau(k), \mu(p_1), V(p_2), \nu_\mu(p_3), \nu_\tau(p_4)$. C_{θ_2} is the angle between μ and τ momenta in the rest frame of $k_2 \equiv p_1 + p_2$ and C_{θ_3} is the angle between k_2 and k in the rest frame of $k_3 \equiv p_1 + p_2 + p_3$. Besides, $\delta_i \equiv \frac{M_i^2}{m_\tau^2}$, $i = \mu, V, 2, 3$ and $M_j^2 \equiv k_j^2$, $j = 2, 3$, ϕ is the angle between the planes composed by $\vec{k}_1 \times \vec{k}_2$ and $\vec{k}_2 \times \vec{k}_3$ and $\lambda(a, b, c) = (a - (\sqrt{b} + \sqrt{c})^2)(a - (\sqrt{b} - \sqrt{c})^2)$.

We point out that the same analysis could be done for $\mu \rightarrow \nu_\mu e \bar{\nu}_e e^+ e^-$, but this is not as restrictive as the constraint (16), due to the smaller phase-space.

D. Electron Anomalous Magnetic Moment

The Standard Model contribution to $a_e = \frac{(g-2)_e}{2}$ has been recently improved up to the tenth order, corresponding to $\Delta a_e = 1159652181.78(77) \times 10^{-12}$ [35], facing the experimental value $a_e = (g-2)/2 = (1159.65218076 \pm 0.00000027) \times 10^{-6}$ [29]. In [36] the author argues that the one-loop correction to this quantity must be reinterpreted as an effective shift of the fine-structure constant, which would not exceed 15 ppb (see eq.(6) of [36]), leading to the following constraint:

$$(e\kappa)^2 I_V \left(\frac{M_V^2}{m_e^2} \right) < 1.5 \times 10^{-8}. \quad (18)$$

The above relation will be considered along with all the bounds presented in the previous subsections.

E. Experimental bounds

There are many experimental searches for the dark sector (see e.g [15, 17, 37–44]). We mention here only the most recent bounds. The NA48/2 collaboration [40] has searched for bound in $\pi^0 \rightarrow \gamma e^+ e^-$ decay and obtained that $\kappa^2 = (0.8 - 1.11) \times 10^{-5}$ at 90% C.L. for the mass of the vector gauge boson in the range $2m_e < M_V < 140$ MeV. The Kloe-2 collaboration determined the bound on the mass of dark photon and photon-dark photon mixing parameter from the study of dark photon contribution in the $\phi \rightarrow \eta V \rightarrow \eta e^+ e^-$ decay, by measuring the cross sections $e^+ e^- \rightarrow V \gamma \rightarrow \mu^+ \mu^- \gamma$ and $e^+ e^- \rightarrow V \gamma \rightarrow e^+ e^- \gamma$. They found that κ^2 has to be smaller then 5×10^{-5} [39, 45].

The BaBar collaboration obtained the very restrictive bounds on the dark Z' boson (corresponds to V in our case) [16] from the cross section for the $e^+ e^- \rightarrow \mu^- \mu^+ Z' \rightarrow \mu^+ \mu^- \mu^+ \mu^-$ process relying on the model described in [46, 47]. A basic feature of this model is the absence of the Z' coupling to the first lepton generation. The BaBar search is based on 514 fb^{-1} of data collected at the PEP-II $e^+ e^-$ storage ring, predominantly taken at the $\Upsilon(4s)$ resonance and their result is applicable also on the models in which the gauge bosons are coupled exclusively to right-handed muons. They obtained a strong bound on the coupling and the mass of Z' in the region $0.2 \text{ GeV} < m_{Z'} < 4 \text{ GeV}$. Since our model contains both dark bosons, dark Higgs and dark gauge boson, we also expect that the inclusion of the dark Higgs contribution in $e^+ e^- \rightarrow \mu^+ \mu^- \mu^+ \mu^-$ might only slightly modify the phenomenology of this channel. Thus, we combine the results of NA48/2, Kloe-2 and BaBar analysis on our plots, which we present in the next section.

F. Discussion

The constraints derived in the previous subsections are presented in Fig. 2. The colored areas are excluded, while the yellow and red bands correspond to the allowed region of the proton charge radius and muon anomalous magnetic moment at 2σ level, respectively. The regions excluded by NA48/2 [40], Kloe-2 [39, 45] and BaBar [16] are grey.

In order to derive a more general conclusion on the proton anomaly explanation, we can combine the definition in (4) along with the constraint in (7) to obtain:

$$\lambda = \frac{3}{2e} \frac{g_\phi^2}{\Delta r_p^2 m_\mu^2}. \quad (19)$$

In addition, by writing the amplitude of $K \rightarrow \mu \nu \phi$ as function of (g_ϕ, m_ϕ) we have concluded that the parameter space for $g_\phi > 0.03$, with $m_\phi < 2m_\mu$, will be ruled out at 90% C.L. The eq.(19) translates this assertion to $\lambda > 0.86e$, a condition that can necessarily exclude the central value of the proton radius discrepancy. We can verify this, for instance, in Fig.2. In (a), even if the above-mentioned limit is respected, the dependence on the small m_ϕ results in the exclusion of the yellow region from the kaon muonic decay. In Fig.2 (b) the $\lambda = 0.8e$ accompanied with a large

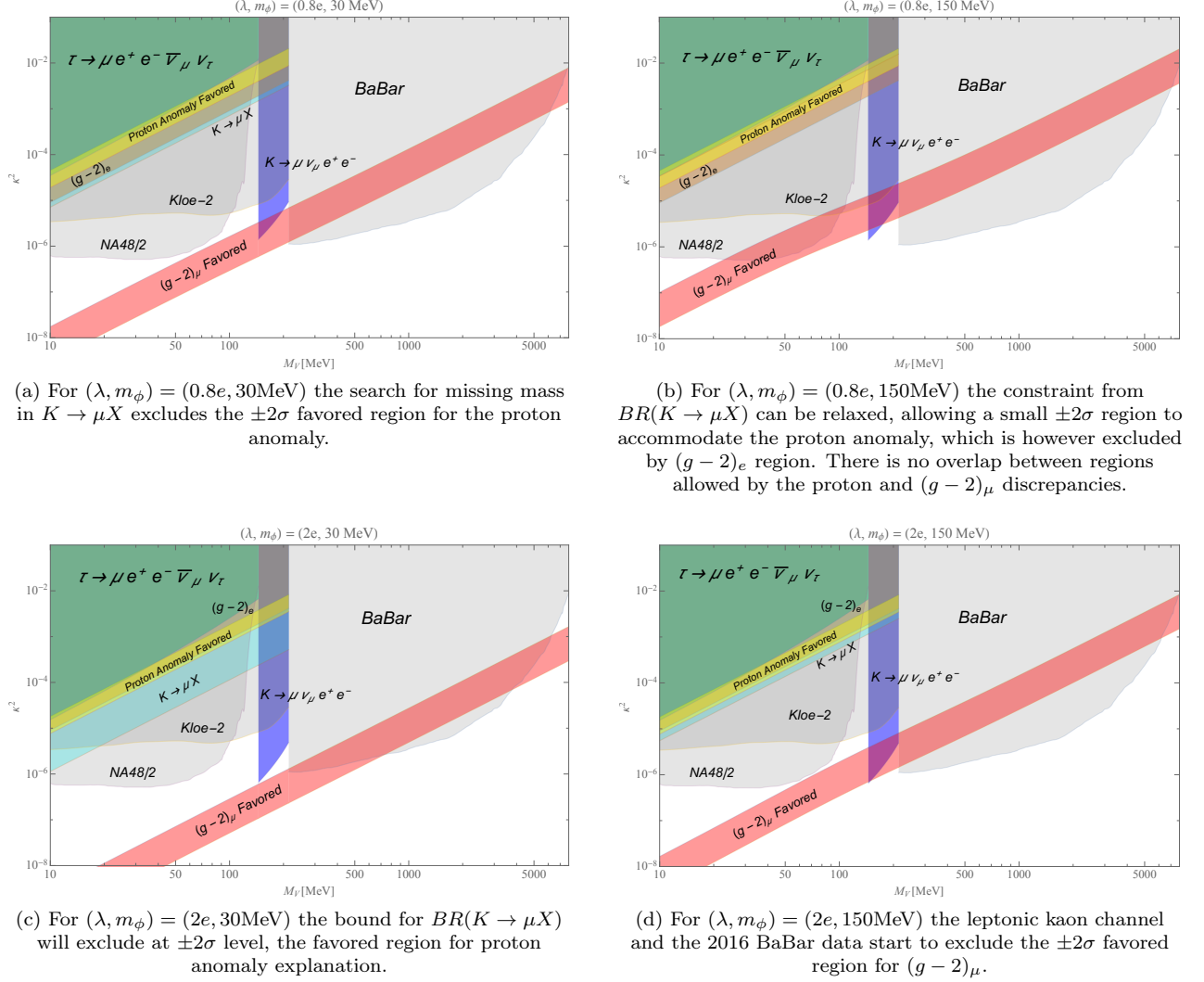


Figure 2: Parameter space for (M_V, κ^2) . The colored regions are excluded by the respective processes and the region favored by $(g-2)_\mu$ at $\pm 2\sigma$ is marked by pink, while the region allowed by the proton size anomaly is yellow.

$m_\phi = 150$ MeV will loose the $K \rightarrow \mu X$ bound. However, in both cases the bound from $(g-2)_e$ will disfavour this sector. In (c) and (d), since $\lambda = 2e$, the proton band is necessarily excluded.

We stress that the only areas in Fig. 2 dependent on m_ϕ are those related to $K \rightarrow \mu X$ and $(g-2)_\mu$. The four plots are pointing out that the proton charge radius cannot be explained by the spontaneously broken dark $U(1)_d$ gauge symmetry. The whole region which allows to explain proton radius puzzle is being excluded by the constraints from $(g-2)_e$, $K \rightarrow \mu X$, $K \rightarrow \mu \nu_\mu e^+ e^-$, and $\tau \rightarrow \mu \nu_\mu e^+ e^-$. In any of the cases we analyse, the proton charge radius anomaly and the muon anomalous magnetic moment cannot be simultaneously explained. Apart from that, the constraint coming from $K \rightarrow \mu \nu_\mu e^+ e^-$ can almost extrapolate the BaBar bound on the mass of vector gauge boson down to $M_V \sim 145$ MeV.

If the size of λ and a large scalar mass are such that the function $F(\lambda, M_V, m_\phi)$ in eq. (10) becomes very small, approaching to zero, the coupling κ tied to the muon anomaly might be arbitrarily large. In Fig.3, for example, the dark Higgs mass is $m_\phi = 250$ MeV and we can find a tiny overlap between the pink and yellow bands. Nevertheless, for $m_\phi > 2m_\mu$ using the constraint $BR(K \rightarrow \mu \nu \mu \mu) < 4.1 \times 10^{-7}$, we find out that it again leads to the exclusion of the proton favoured region at 90% C.L.

We finally note that the contribution of both the dark Higgs and the dark V can enable $(g-2)_\mu$ to be explained. If, for example, only the dark gauge boson is present, there would be no region on the parameter space allowed by experimental results - and by the bounds we have shown here - that could explain the respective anomaly.

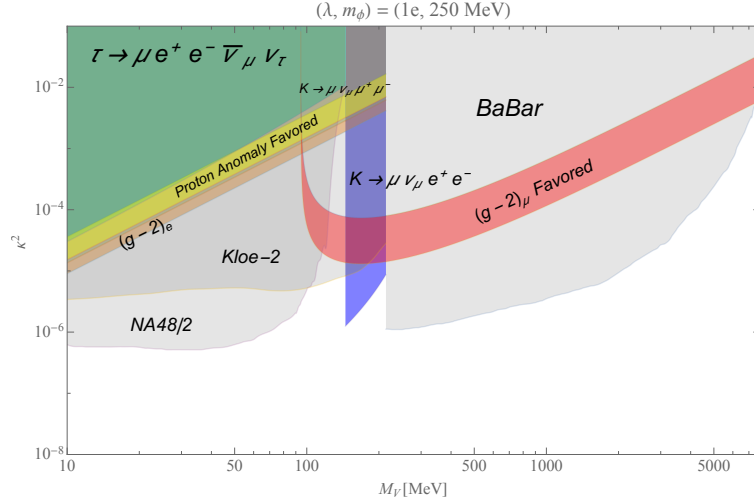


Figure 3: Parameter space for (M_V, κ^2) . The colored regions are excluded by the processes denoted on the respective areas. The region favored by $(g-2)_\mu$ at $\pm 2\sigma$ is marked by red and proton radius anomaly by yellow color. The size of λ and the large scalar mass are such that $F(\lambda, M_V, m_\phi)$ in eq. (10) goes to zero, requiring large values of κ . For $m_\phi > 2m_\mu$ we use the constraint from $BR(K \rightarrow \mu\nu\mu\mu) < 4.1 \times 10^{-7}$ which excludes the proton favored region at 90% C.L.

III. PREDICTIONS AT LOW ENERGIES

Our analyses of the dark $U(1)_d$ gauge sector allows the mass of M_V to be in the region around $50 < M_V(\text{MeV}) < 150$ with the parameter $\kappa \sim 10^{-3}$. One would expect that the weak decays are more likely to offer good testing ground for the dark sector [48]. Particularly, the flavor changing neutral current processes occurring in meson decays were most favorable for such searches. For example in ref. [49] the rare decay of K and B mesons to $\pi e^+ e^-$ were suggested as interesting candidates for the dark boson searches, mainly due to the low rate of $BR(K^+ \rightarrow \pi^+ e^+ e^-)_{exp} = (3.00 \pm 0.09) \times 10^{-7}$ in the SM. By relying on the reanalysis of $K \rightarrow \pi\gamma^*$ in [1], we calculate the branching ratio for $K \rightarrow \pi V$ and present our result in Fig. 4. The V dark boson promptly decays to $V \rightarrow e^+ e^-$ and the narrow-width approximation will then give that $BR(K \rightarrow \pi V \rightarrow \pi e^+ e^-) = BR(K \rightarrow \pi V)$.

A. $K \rightarrow \pi V$

In the eq. (12) of [36], the author presented a general formula for the branching ratio of $K \rightarrow \pi V$ valid for M_V below 200 MeV and given by:

$$Br_{K \rightarrow \pi V} \simeq 8 \times 10^{-5} \times \kappa^2 \left(\frac{M_V}{100 \text{ MeV}} \right)^2 \quad (20)$$

If we replace κ^2 by the value which explains $(g-2)_\mu$ anomaly in eq. (10), we can derive the branching ratio as a function of M_V for a specific choice of the parameters (λ, m_ϕ) . Some examples are presented in Fig. 4.

The NA48/2 collaboration has commented in ref. [40] that sensitivity on this process is not competitive with the existing bounds. Namely, they found that $\kappa^2 = (0.8 - 1.11) \times 10^{-5}$ at 90% CL. for the vector gauge boson mass in the range $2m_e < M_V < 140 \text{ MeV}$.

B. $e^+ e^- \rightarrow \mu^+ \mu^- (\phi \rightarrow \mu^+ \mu^-)$ and $e^+ e^- \rightarrow \mu^+ \mu^- (V \rightarrow e^+ e^-)$

In the work of ref. [16] the search for a direct production of muonic dark forces in a model-independent method was done. The results were presented as the measured $e^+ e^- \rightarrow \mu^+ \mu^- Z', Z' \rightarrow \mu^+ \mu^-$ cross-section being a function of the Z' mass. Within the dark $U(1)_d$ model we consider in this paper, the only contribution to the process $e^+ e^- \rightarrow \mu^+ \mu^- \mu^+ \mu^-$ is from the dark Higgs scalar ϕ in the region $M_V < 2m_\mu$ and it is presented in Fig. 5(a).

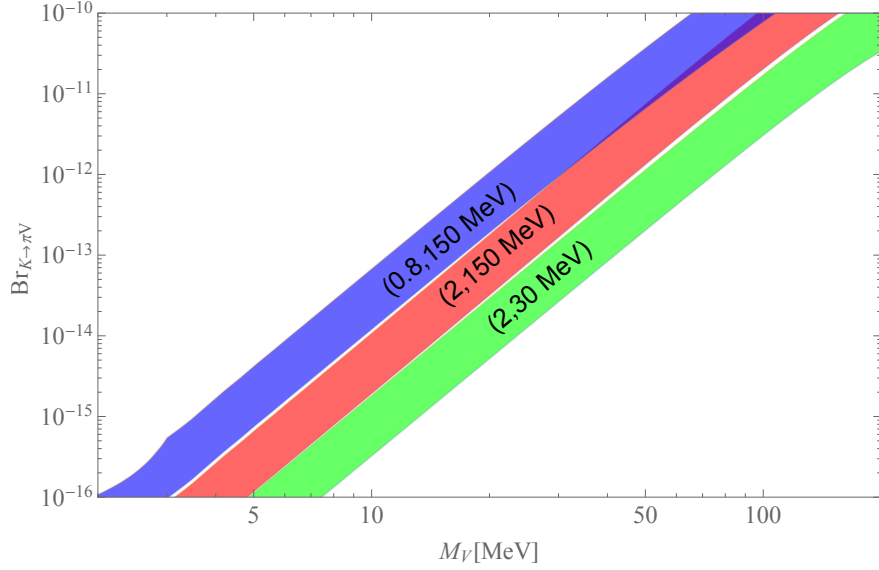
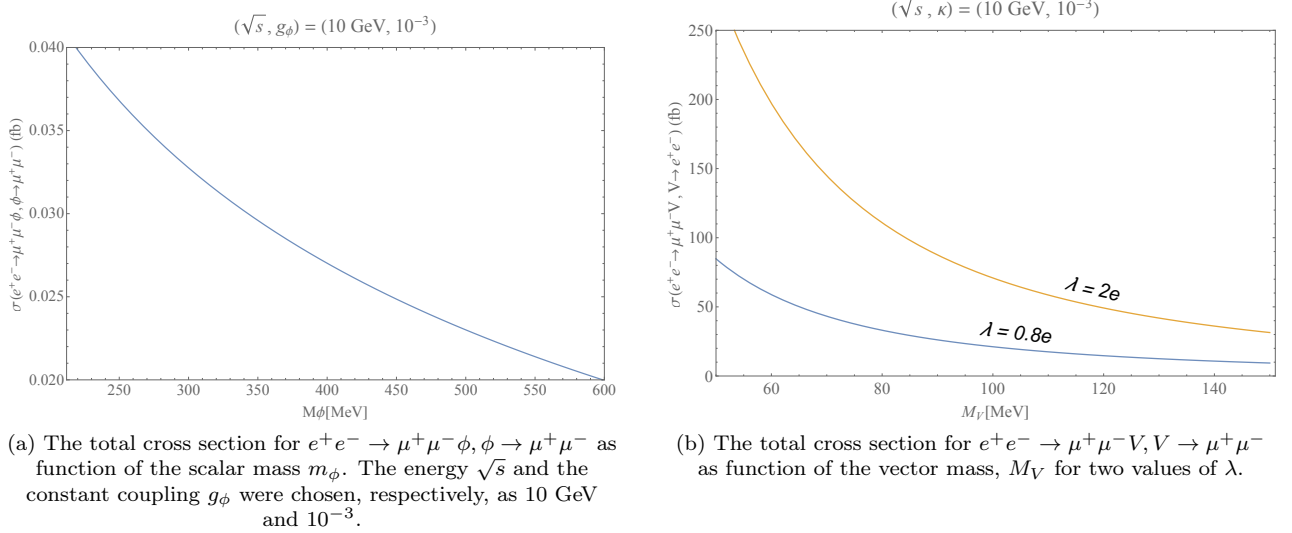


Figure 4: The branching ratio of $K \rightarrow \pi V$ for some specific parameters (λ, m_ϕ) . The values of the coupling κ are given by the favored region of $(g - 2)_\mu$ up to 2σ . In this model V decays promptly to $V \rightarrow e^+e^-$.



(a) The total cross section for $e^+e^- \rightarrow \mu^+\mu^-\phi, \phi \rightarrow \mu^+\mu^-$ as function of the scalar mass m_ϕ . The energy \sqrt{s} and the constant coupling g_ϕ were chosen, respectively, as 10 GeV and 10^{-3} .

(b) The total cross section for $e^+e^- \rightarrow \mu^+\mu^-V, V \rightarrow \mu^+\mu^-$ as function of the vector mass, M_V for two values of λ .

Figure 5: The total cross section for $e^+e^- \rightarrow \mu^+\mu^-\mu^+\mu^-$ in the framework of ϕ and V emission. The results are complementary to the Fig. 3 of [16].

Nevertheless, a complementary search at a low-mass region for V will be in the process $e^+e^- \rightarrow \mu^+\mu^-V, V \rightarrow e^+e^-$. The theoretical results are presented in Fig. 5(b) for the center-of-mass energy equal to $\sqrt{10}$ GeV.

C. $\pi^0 \rightarrow \gamma e^+e^-$ and $\eta \rightarrow \gamma e^+e^-$

The dark $U(1)_d$ sector might affect low energy observables due to the mixing of the SM photon with the part of dark gauge boson. That means in all processes where this conversion $\gamma \leftrightarrow V$ occurs, one can search for the dark boson. Due to the long lived vector, the use of narrow-width approximation is fully justified and we use $Br(V \rightarrow e^+e^-) = 1$. As already suggested in ref. [50] the search for the presence of dark sector in electromagnetic decays seems to be possible. We make predictions for the decays of $P \rightarrow \gamma V \rightarrow \gamma e^+e^-$ for $P = \pi^0, \eta$ by noticing that our V can have the mass in the region $50 \text{ MeV} < M_V < 150 \text{ MeV}$ and can decay only to the electron-positron pair. In ref. [50]

$\pi^0(\eta) \rightarrow \gamma V \rightarrow \gamma e^+ e^-$ were approached by relying on the result of model independent study given in ref. [51]. In this approach the branching ratio of $\pi^0 \rightarrow \gamma e^+ e^-$ agrees very well with the experimental one. The decay width for $\pi^0 \rightarrow \gamma V \rightarrow \gamma e^+ e^-$ with the use of the narrow-width approximation can be written as:

$$\Gamma(\pi^0 \rightarrow \gamma V \rightarrow \gamma e^+ e^-) = |f(1, 0, x_V)|^2 \Gamma(\pi^0 \rightarrow \gamma V) Br(V \rightarrow e^+ e^-), \quad (21)$$

with $\Gamma(\pi^0 \rightarrow \gamma V) = 2\kappa^2(1 - M_V^2/m_\pi^2)^3 \Gamma_0$, $\Gamma_0 \equiv \Gamma(\pi^0 \rightarrow \gamma \gamma)$ and $f(1, 0, x) = 1 + a_\pi x + b_\pi x^2 + \mathcal{O}(x^3)$, with $a_\pi = -0.0324(12)_{stat}(19)_{sys}$, $b_\pi = 1.06(9)_{stat}(25)_{sys} \times 10^{-3}$ and $x_V = M_V^2/m_{\pi^0}^2$, as explained in details in [50]. For the value $M_V = 50$ MeV, we obtained $BR(\pi^0 \rightarrow \gamma V \rightarrow \gamma e^+ e^-) = 1.3 \times 10^{-6}(\kappa/10^{-3})^2$, while for the $M_V = 100$ MeV we calculate $BR(\pi^0 \rightarrow \gamma V \rightarrow \gamma e^+ e^-) = 1.9 \times 10^{-7}(\kappa/10^{-3})^2$.

For the decay of $\eta \rightarrow \gamma V \rightarrow \gamma e^+ e^-$ following [50] and using the transition form-factor from the same reference $f_\eta(1, 0, x) = 1 + b_\eta x + c_\eta x^2 + d_\eta x^3 \mathcal{O}(x^4)$ ($b_\eta = 0.576(11)_{stat}(4)_{sys}$, $c_\eta = 0.339(15)_{stat}(5)_{sys}$ and $d_\eta = 0.200(14)_{stat}(18)_{sys}$, and $x_V = M_V^2/m_\eta^2$ [52]), we obtain for $M_V = 50$ MeV the branching ratio $BR(\eta \rightarrow \gamma V \rightarrow \gamma e^+ e^-) = 1.5 \times 10^{-6}(\kappa/10^{-3})^2$, while for $M_V = 100$ MeV we calculate $BR(\eta \rightarrow \gamma V \rightarrow \gamma e^+ e^-) = 1.4 \times 10^{-6}(\kappa/10^{-3})^2$. There are few experimental studies of the dark matter contributions in $\pi^0(\eta) \rightarrow \gamma e^+ e^-$ as described in [50] starting with the beam-dump experiments E141 [53], CHARM [54], NA48/2 [40]. Also, there are plans for the future facilities as APEX [15], HPS [55] DarkLight [56] and LHCb[57]. The NA48/2 experiment almost reached sensitivity on the mixing parameter $\kappa \sim 10^{-3}$ [40] in their search for the dark photons in $\pi^0(\eta) \rightarrow \gamma e^+ e^-$. In [54], based on NOMAD and PS191 it was claimed that the bound on $BR(\pi^0 \rightarrow \gamma X \rightarrow \gamma e^+ e^-) \leq 10^{-15}$ can be reached and for the decay $BR(\eta \rightarrow X \gamma \rightarrow \gamma e^+ e^-) \leq 10^{-14}$ [58].

D. $\rho \rightarrow \pi e^+ e^-$, $K^* \rightarrow K e^+ e^-$ and $\phi(1020) \rightarrow \eta e^+ e^-$

The amplitude for the decays $P^* \rightarrow PV$, $P^* = \rho^{+,0}$, $K^{*,0}$, ϕ and $P = \pi^{+,0}$, $K^{+,0}$ and η can be written as:

$$\mathcal{M}(P^*(p_{P^*}, \epsilon_{P^*}) \rightarrow P(p_P)V(p_V, \epsilon_V) = \kappa g_{P^*PV} \epsilon_{\mu\nu\alpha\beta} p_{P^*}^\mu \epsilon_{P^*}^\nu p_V^\alpha \epsilon_V^\beta, \quad (22)$$

with p_P , p_{P^*} and p_V being the momenta of the corresponding mesons and ϵ_{P^*} and ϵ_V being polarization vectors of P^* and V , respectively. In order to determine decay widths, we assume that to a good approximation $g_{P^*PV} \simeq g_{P^*P\gamma}$. We expect that this approximation is satisfied as long as the dark vector boson mass is relatively small. There are numerous attempts within lattice QCD community to calculate $\rho\pi\gamma^*$ form-factors [59] which will help in more precise studies of g_{P^*PV} . The transition coefficient $g_{P^*P\gamma}$ can be extracted from the decay width for $P^* \rightarrow P\gamma$. Knowing that $\Gamma(P^* \rightarrow P\gamma) = |g_{P^*P\gamma}|^2(m_{P^*}^2 - m_P^2)^3/(96\pi m_{P^*}^3)$, one can determine $g_{P^*P\gamma}$. It was found by the authors of [60] that $g_{\rho^+\pi^+\gamma} = 2.19 \times 10^{-4} \text{ MeV}^{-1}$, $g_{\rho^0\pi^0\gamma} = 2.52 \times 10^{-4} \text{ MeV}^{-1}$, $g_{K^{*+}K^+\gamma} = 2.53 \times 10^{-4} \text{ MeV}^{-1}$, $g_{K^{*0}K^0\gamma} = 2.19 \times 10^{-4} \text{ MeV}^{-1}$ and we obtain the value $g_{\Phi(1020)\eta\gamma} = 1.26 \times 10^{-4} \text{ MeV}^{-1}$, using data given in PDG [29]. The decay width for $P^* \rightarrow PV$ can be written as:

$$\Gamma(P^* \rightarrow PV) = \frac{|\kappa g_{P^*PV}|^2}{96\pi} \frac{\lambda(m_{P^*}^2, m_P^2, M_V^2)^{3/2}}{m_{P^*}^3}, \quad (23)$$

and finally:

$$\Gamma(P^* \rightarrow PV \rightarrow P e^+ e^-) = \Gamma(P^* \rightarrow PV) BR(V \rightarrow e^+ e^-). \quad (24)$$

Table I: Predicted branching ratios for $BR(P^* \rightarrow PV \rightarrow P e^+ e^-)$ for the dark gauge boson mass $M_V = 50, 100$ MeV and $\kappa = 0.001$. For other values of the photon-V mixing parameter κ one should rescale these results by $(\kappa/10^{-3})^2$.

P^*, P, V	$M_V = 50 \text{ MeV}$	$M_V = 100 \text{ MeV}$
ρ^0, π^0, V	6.3×10^{-10}	6.1×10^{-10}
ρ^+, π^+, V	4.8×10^{-10}	4.6×10^{-10}
K^{*0}, K^0, V	7.6×10^{-10}	7.0×10^{-10}
K^{*+}, K^+, V	1.0×10^{-9}	9.5×10^{-10}
$\Phi(1020), \eta, V$	9.1×10^{-10}	8.9×10^{-10}

There are a number of planned experimental searches in which above-mentioned processes might be relevant as APEX [15], HPS [55] DarkLight [56] and LHCb[57]. The KLOE-2 experiment has already searched for the dark photon contribution in $\phi \rightarrow \eta V \rightarrow V e^+ e^-$ decay [43] not finding any bump in the differential distribution.

IV. SUMMARY

The $U(1)_d$ gauge model of eq.(1) introduces a new dark gauge boson and a dark Higgs. The model was first proposed to explain the proton charge radius discrepancy as well as the $(g-2)_\mu$ anomaly.

We have explored the phenomenology of these two dark bosons - the dark Higgs ϕ and the vector V - through a set of low energy processes, focusing on the parameter space (M_V, κ) . In our approach the muon magnetic moment receives the contribution of both particles and the ϕ mass provides an additional freedom to adjust the allowed band of $(g-2)_\mu$ within 2σ . We find out that V has to decay to e^+e^- in order to explain $K \rightarrow \mu X$, where X refers to a missing energy, implying that in this process any signature of the dark Higgs could be detected. Further, we concluded that the allowed band for the proton radius anomaly is strongly constrained by a set of well-established bounds, namely the bounds from $(g-2)_e$ and $\tau \rightarrow \nu_\tau \mu \nu_\mu e^+ e^-$ decay. This feature, for instance, will enable different ranges for M_V which were at first excluded in the context of a generic vector coupling by the BaBar searches [16], as presented in Fig. 2(c). The bounds from $K \rightarrow \mu \nu_\mu e^+ e^-$, $K \rightarrow \mu \nu_\mu \mu^+ \mu^-$, $\tau \rightarrow \nu_\tau \mu \nu_\mu e^+ e^-$ when combined with above-mentioned bounds allow the mass of M_V to be in the region around $50 \text{ MeV} < M_V < 150 \text{ MeV}$ with the parameter $\kappa \sim 10^{-3}$, while the mass of the dark Higgs can be from few MeV till $\sim 200 \text{ MeV}$, for a particular choice of the remaining parameter λ . We finally mention that the bound from $K \rightarrow \mu \nu_\mu e^+ e^-$ leads to constraints as strong as the experimentally achieved by the BaBar and NA48/2 collaborations on the correspondent sector.

We have also presented a set of predictions. The very small branching ratios of the processes $K \rightarrow \pi V, V \rightarrow e^+ e^-$, as it was pointed in [36], makes the search for the dark gauge boson rather difficult. The electromagnetic decays of $\pi^0 \rightarrow \gamma e^+ e^-$ and $\eta \rightarrow \gamma e^+ e^-$, $\rho \rightarrow \pi e^+ e^-$, $K^* \rightarrow K e^+ e^-$ and $\phi(1020) \rightarrow \eta e^+ e^-$ might also proceed through the dark gauge boson. Some of these processes are already subjects of experimental studies. The small mixing parameter between the photon and dark-photon suppresses the branching ratios for these processes, but hopefully future experiments for the dark matter search would shed more light on dark bosons at low energies.

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